

# Generation of field mediated three qubit entangled state shared by Alice and Bob

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A scheme to generate shared tripartite entangled states, with two-trapped atoms in a cavity held by Alice (qubits  $A_1$  and  $A_2$ ) entangled to a single trapped atom in a remote lab owned by Bob ( $B$ ), is proposed. The entanglement is generated through interaction of trapped atoms with two mode squeezed light shared by the two cavities. The proposed scheme is an extension of the proposal of ref. [W. Son, M. S. Kim, J. Lee, and D. Ahn, J. Mod. Opt. 49, 1739 (2002)], where the possibility of entangling two remote qubits using a bipartite continuous variable state was examined. While the global negativity detects the free entanglement of the three atom mixed state, the bound entanglement is detected by the negativity calculated from pure state decomposition of the state operator. The partial negativities calculated by selective partial transposition of the three atom mixed state detect the pairwise entanglement of qubit pairs  $A_1B$ ,  $A_2B$ , and  $A_1A_2$ . The entanglement of three atoms is found to be W-like, no GHZ like quantum correlations being generated.

## I. INTRODUCTION

Quantum systems with discrete energy spectrum are often used for storing, manipulating, and transmitting information encoded in quantum states belonging to finite-dimensional Hilbert spaces. A two-level atom is an example of two-dimensional quantum systems, universally known as qubits. All possible superposition states of a two-level system can be expressed in terms of linear combinations of two-dimensional orthonormal states defining the standard computational basis. On the other hand, the electromagnetic field is characterized by continuous variables that is observables with a continuous spectrum of eigenvalues. Quantum information based on continuous variables is encoded in quantum states belonging to infinite-dimensional Hilbert spaces. The entanglement of remote qubits is a resource for implementation of communication protocols [1] and information processing [2] using quantum systems. W. Son et al. [3] have shown the possibility of entangling two remote qubits using a bipartite continuous variable state. In this article, a scheme to generate shared tripartite entangled states of two-trapped atoms in a cavity held by Alice (qubits  $A_1$  and  $A_2$ ) and a single trapped atom in a remote lab owned by Bob ( $B$ ), is proposed. A single mode of the two-mode squeezed electromagnetic field, generated by a squeezed light source, is injected into each cavity by a beam splitter. The entanglement is generated through interaction of trapped atoms with quantized electromagnetic field shared by the two cavities. No direct interaction amongst the atoms takes place.

The dynamics of three atom entanglement due to atom field interaction is investigated after tracing over the field degrees of freedom. We use the global negativity and partial  $K$ -way negativities to analyze the entanglement dynamics. Negativity [4], based on Peres Horodecki [5, 6] criterion, has been shown to be an entanglement monotone [7]. A comparison of global negativity of three atom mixed state with respect to qubit  $B$  and the linear entropy calculated from the state operator for qubit  $B$  indicates that for certain ranges of interaction parameter values the global negativity fails to detect the entanglement amongst the atomic qubits. Using pure state decomposition (PSD) of mixed three atom state, we recalculate the negativity by making a weighted sum of pure states contributions to global negativity. The global negativity calculated from pure state decomposition (PSDG) of the state operator is not a proper measure of entanglement because it is not invariant with respect to local operations and classical communication (LOCC). It does, however, detect the bound entanglement being greater or equal to the convex roof extension (CRE) of negativity shown to be an entanglement monotone [8]. We use the global negativity and partial  $K$ -way negativities of pure state decomposition, primarily, to understand how quantum coherences present in the three atom state, evolve with interaction time. The coherences of a multipartite composite system having  $N$  subsystems can be quantified by partial  $K$ -way ( $2 \leq K \leq N$ ) negativities [9, 10]. Partial  $K$ -way negativity is the contribution of a  $K$ -way partial transpose to global negativity. In a three qubit state, the entanglement arising due to 3-way and 2-way coherences [11] is quantified by partial 3-way and 2-way negativities. For canonical states, positive valued partial 3-way and 2-way negativities measure, respectively, the three qubit GHZ-like and pairwise entanglement of the system [12]. We further use the partial negativities calculated from the pure state decomposition

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of three atom mixed state to study the dynamics of two-body coherences of qubit pairs  $A_1B$ ,  $A_2B$ , and  $A_1A_2$ . The qubits  $A_1$  and  $A_2$  are shown to form a bell pair with certain probability, which is entangled to qubit  $B$ . The degree of entanglement depends on the atom field coupling strength, squeeze parameter, as well as the coefficient of reflection of the beam splitter.

## II. THE CAVITY FIELD

Consider two cavities prepared in the vacuum state at  $t = 0$ . The beam splitter used to inject the external field into a cavity is represented by the operator

$$\hat{B}(\theta) = \exp \left[ \frac{\theta}{2} (\hat{c}\hat{f}^\dagger - \hat{c}^\dagger\hat{f}) \right], \quad (1)$$

where  $\hat{c}$  ( $\hat{f}$ ) and  $\hat{c}^\dagger$  ( $\hat{f}^\dagger$ ) are creation and annihilation operators for photons in the cavity (external field), respectively. The coefficient  $r = \cos \frac{\theta}{2}$  is the reflection coefficient of the beam splitter. The two mode squeeze operator acting on two mode vacuum state

$$\hat{S}(s) |0, 0\rangle = \exp(-s\hat{a}\hat{b} + s\hat{a}^\dagger\hat{b}^\dagger) |0, 0\rangle, \quad (2)$$

produces a two-mode squeezed field in composite state

$$|\Psi_F\rangle = \frac{1}{\cosh s} \sum_{n=0}^{\infty} (\tanh s)^n |n, n\rangle, \quad (3)$$

where  $s$  is the squeeze parameter. The bosonic creation and annihilation operators for mode one are  $\hat{a}^\dagger$ ,  $\hat{a}$  and for mode two are  $\hat{b}^\dagger$ ,  $\hat{b}$ , respectively. The two mode squeezed state is an entangled state having bipartite entanglement determined by the value of squeeze parameter  $s$ . Using beam splitter (represented by operator of Eq. (1)) to inject the modes one and two of the squeezed field (Eq. (3)) into independent cavities  $c_1$  and  $c_2$ , respectively, the cavity field state at  $t = 0$  reads as

$$\hat{\rho}_{c_1 c_2}(0) = \left( \frac{1}{\cosh s} \right)^2 \sum_{n,m=0}^{\infty} \sum_{k,l=0}^{\min[n,m]} (\tanh s)^{n+m} G_{kl}^{mm}(\theta) |n-k, n-l\rangle \langle m-k, m-l|, \quad (4)$$

where

$$G_{kl}^{mn}(\theta) = C_k^n(\theta) C_k^m(\theta) C_l^n(\theta) C_l^m(\theta), \quad (5)$$

$$G_{kl}^{nn}(\theta) = \frac{n!}{k!(n-k)!} \frac{n!}{l!(n-l)!} \cos^{2k} \left( \frac{\theta}{2} \right) \sin^{2n-2k} \left( \frac{\theta}{2} \right) \cos^{2l} \left( \frac{\theta}{2} \right) \sin^{2n-2l} \left( \frac{\theta}{2} \right), \quad (6)$$

and

$$C_k^n(\theta) = \sqrt{\frac{n!}{k!(n-k)!}} \cos^k \left( \frac{\theta}{2} \right) \sin^{n-k} \left( \frac{\theta}{2} \right). \quad (7)$$

The operator  $\hat{\rho}_{c_1 c_2}(0)$  represents a mixed composite state of cavity field.

## III. ATOM FIELD INTERACTION

The interaction of  $N$  two-level atoms with a resonant single mode quantized electromagnetic field in rotating wave approximation is described by Tavis -Cummings model [13] Hamiltonian

$$\hat{H}_N = \hbar\omega_0 \hat{a}^\dagger \hat{a} + \frac{\hbar\omega_0}{2} \sum_{i=1,N} \hat{\sigma}_z^i + \hbar g \sum_{i=1,N} [\hat{\sigma}_+^i \hat{a} + \hat{\sigma}_-^i \hat{a}^\dagger], \quad (8)$$

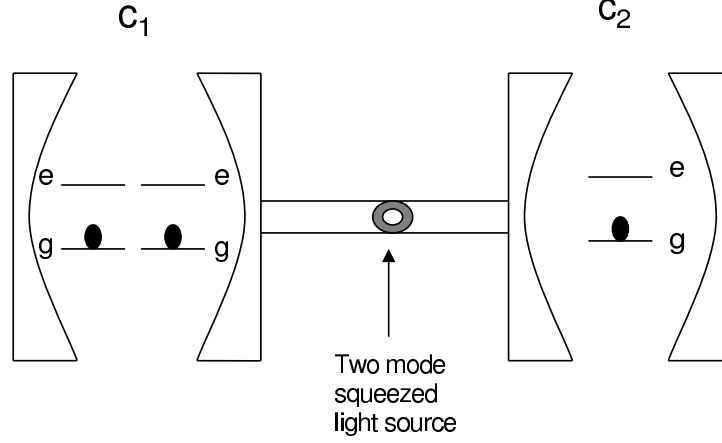


FIG. 1: The schematic diagram of experimental setup

where  $\omega_0$  is the atomic transition frequency and  $g$  the atom field coupling strength. Eigen states of Pauli operator  $\hat{\sigma}_z^i$  model the internal states of the  $i^{th}$  atom ( $i = 1, 2$ ) with eigenvalue  $m_\sigma^i = -1$  ( $+1$ ) standing for the ground (excited) state of the atom. Defining the composite operator,  $\hat{\sigma}_k = \sum_i \hat{\sigma}_k^i$  where  $k = (z, +, -)$ , we may construct the eigen basis of operators  $\hat{\sigma}^2$  and  $\hat{\sigma}_z$  to represent  $N$  atom internal states. A typical coupled basis vector is written as  $|\sigma, m_\sigma\rangle$ , with eigenvalues of  $\hat{\sigma}^2$  and  $\hat{\sigma}_z$  given by  $\sigma(\sigma + 2)$  and  $m_\sigma$ , respectively.

Consider two atoms trapped in cavity  $c_1$  radiated by the single mode resonant cavity field in photon number state  $|n\rangle$ . Interaction with the cavity field generates entanglement of internal states of the atom, and the cavity field state. Working in the atom-field basis  $|\sigma, m_\sigma, n\rangle$ , the relevant basis states for the case when two atoms are prepared in their ground states at  $t = 0$  are  $|2, -2, n\rangle$ ,  $|2, 0, n-1\rangle$ , and  $|2, 2, n-2\rangle$  and the Hamiltonian matrix reads as

$$H_{1I}^n = \begin{pmatrix} 0 & \hbar g B_n & 0 \\ \hbar g B_n & 0 & \hbar g A_n \\ 0 & \hbar g A_n & 0 \end{pmatrix}, \quad (9)$$

where  $A_n = \sqrt{2(n-1)}$ ,  $B_n = \sqrt{2n}$ . Defining interaction parameter  $\tau = gt$ , the unitary operator  $\widehat{U}_1(\tau) = \exp[-(i\widehat{H}_{1I}t)/\hbar]$  is found to be

$$U_1^n(\tau) = \begin{pmatrix} \frac{[B_n^2 \cos(f_n \tau) + A_n^2]}{A_n^2 + B_n^2} & \frac{-iB_n \sin(f_n \tau)}{\sqrt{(A_n^2 + B_n^2)}} & \frac{A_n B_n [\cos(f_n \tau) - 1]}{A_n^2 + B_n^2} \\ \frac{-iB_n \sin(f_n \tau)}{\sqrt{(A_n^2 + B_n^2)}} & \cos(f_n \tau) & \frac{-iA_n \sin(f_n \tau)}{\sqrt{(A_n^2 + B_n^2)}} \\ \frac{A_n B_n [\cos(f_n \tau) - 1]}{A_n^2 + B_n^2} & -iA_n \sin(f_n g \tau) & \frac{[A_n^2 \cos(f_n \tau) + B_n^2]}{A_n^2 + B_n^2} \end{pmatrix}, \quad (10)$$

where  $f_n = \sqrt{2(2n-1)}$ .

For a single atom interacting with the cavity field in cavity  $c_2$ , the unitary matrix that determines the time evolution of the system in the basis  $|1, -1, m\rangle, |1, 1, m-1\rangle$  is

$$U_2^m(\tau) = \begin{pmatrix} \cos(\sqrt{m}\tau) & -i \sin(\sqrt{m}\tau) \\ i \sin(\sqrt{m}\tau) & \cos(\sqrt{m}\tau) \end{pmatrix}. \quad (11)$$

The evolution operator  $\hat{U}_{12}^{nm}(\tau)$  for the two cavity composite system is obtained by taking the tensor product

$$\hat{U}_{12}^{nm}(\tau) = \hat{U}_1^n(\tau) \otimes \hat{U}_2^m(\tau). \quad (12)$$

#### IV. THREE QUBIT STATE OPERATOR

Consider two-trapped atoms in cavity  $c_1$  held by Alice ( qubits  $A_1$  and  $A_2$ ) and a single trapped atom (qubit  $B$ ) in a remote cavity  $c_2$  controlled by Bob. The atoms are cooled down to their ground state with cavities prepared in vacuum state at  $t = 0$ . A squeezed light source generates the two mode electromagnetic field. A schematic diagram of the setup is shown in figure(1). The two mode squeezed field is injected into the cavities, with each cavity receiving a single mode of the field. The composite state of field in two cavities is given by  $\hat{\rho}_{c_1 c_2}(0)$  as in Eq. (4). The state of the composite system without atom field interaction is represented by

$$\hat{\rho}_{(0)} = \hat{\rho}^{A_1 A_2 B}(0) \otimes \hat{\rho}_{c_1 c_2}(0), \quad (13)$$

where  $\hat{\rho}^{A_1 A_2 B}(0) = |2, -2\rangle_{c_1} |1, -1\rangle_{c_2} \langle 1, -1|_{c_1} \langle 2, -2|_{c_2}$  in the coupled basis (basis states labelled by eigenvalues of  $\hat{\sigma}^2$  and  $\hat{\sigma}_z$ ). Here  $|2, -2\rangle_{c_1}$  represents qubits  $A_1$  and  $A_2$  in state  $|2, -2\rangle$  in cavity  $c_1$ . The state operator, after interaction between the atoms and the respective squeezed fields in the two cavities for a time  $t = \tau/g$ , is given by

$$\hat{\rho}(\tau) = \hat{U}_{12}(\tau) \hat{\rho}^{A_1 A_2 B}(0) \otimes \hat{\rho}_{c_1 c_2}(0) \hat{U}_{12}^\dagger(\tau) \quad (14)$$

Using the unitary evolution operators due to atom-field interaction in cavities one and two (Eq. (12)), the state of the composite system after interaction time  $t$  is written as

$$\begin{aligned} \hat{\rho}(\tau) &= \left( \frac{1}{\cosh s} \right)^2 \hat{U}_{12}(\tau) \left[ \sum_{n,m=0}^{\infty} (\tanh s)^{n+m} \right. \\ &\quad \left. \sum_{k,l=0}^{\min[n,m]} G_{kl}^{nm}(\theta) |2, -2, n-k\rangle |1, -1, n-l\rangle \langle 2, -2, m-k| \langle 1, -1, m-l| \right] \hat{U}_{12}^\dagger(\tau) \\ &= \left( \frac{1}{\cosh s} \right)^2 \sum_{n,m=0}^{\infty} \sum_{k,l=0}^{\min[n,m]} (\tanh s)^{n+m} G_{kl}^{nm}(\theta) \left\{ \hat{U}_1^{n-k}(\tau) \otimes \hat{U}_2^{n-l}(\tau) \right. \\ &\quad \left. |2, -2, n-k\rangle |1, -1, n-l\rangle \langle 2, -2, m-k| \langle 1, -1, m-l| \hat{U}_1^{\dagger m-k}(\tau) \otimes \hat{U}_2^{\dagger m-l}(\tau) \right\}. \end{aligned} \quad (15)$$

The reduced state operator for atoms is obtained from  $\hat{\rho}(\tau)$ , by tracing over the field modes, that is

$$\hat{\rho}^{A_1 A_2 B}(\tau) = \text{Tr}_{c_1 c_2} (\hat{\rho}(\tau)). \quad (16)$$

Associating, single qubit states  $|0\rangle$  ( $|1\rangle$ ) with ground (excited) state of the two-level atom, the initial state of three atoms in computational basis reads as  $\hat{\rho}^{A_1 A_2 B}(0) = (|00\rangle_{c_1} \langle 00|) \otimes (|0\rangle_{c_2} \langle 0|)$ . The matrix  $\hat{\rho}^{A_1 A_2 B}(\tau)$  in the computational basis  $|00\rangle_{c_1} |0\rangle_{c_2}$ ,  $|10\rangle_{c_1} |0\rangle_{c_2}$ ,  $|01\rangle_{c_1} |0\rangle_{c_2}$ ,  $|11\rangle_{c_1} |0\rangle_{c_2}$ ,  $|00\rangle_{c_1} |1\rangle_{c_2}$ ,  $|10\rangle_{c_1} |1\rangle_{c_2}$ ,  $|01\rangle_{c_1} |1\rangle_{c_2}$ ,  $|11\rangle_{c_1} |1\rangle_{c_2}$ , reads as

$$\hat{\rho}^{A_1 A_2 B}(\tau) = \begin{pmatrix} \rho_{11}^{A_1 A_2 B} & 0 & 0 & 0 & 0 & \frac{\rho_{15}^{A_1 A_2 B}}{\sqrt{2}} & \frac{\rho_{15}^{A_1 A_2 B}}{\sqrt{2}} & 0 \\ 0 & \frac{\rho_{22}^{A_1 A_2 B}}{2} & \frac{\rho_{22}^{A_1 A_2 B}}{2} & 0 & 0 & 0 & 0 & \frac{\rho_{26}^{A_1 A_2 B}}{\sqrt{2}} \\ 0 & \frac{\rho_{22}^{A_1 A_2 B}}{2} & \frac{\rho_{22}^{A_1 A_2 B}}{2} & 0 & 0 & 0 & 0 & \frac{\rho_{26}^{A_1 A_2 B}}{\sqrt{2}} \\ 0 & 0 & 0 & \rho_{33}^{A_1 A_2 B} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_{44}^{A_1 A_2 B} & 0 & 0 & 0 \\ \frac{\rho_{15}^{A_1 A_2 B}}{\sqrt{2}} & 0 & 0 & 0 & 0 & \frac{\rho_{55}^{A_1 A_2 B}}{2} & \frac{\rho_{55}^{A_1 A_2 B}}{2} & 0 \\ \frac{\rho_{15}^{A_1 A_2 B}}{\sqrt{2}} & 0 & 0 & 0 & 0 & \frac{\rho_{55}^{A_1 A_2 B}}{2} & \frac{\rho_{55}^{A_1 A_2 B}}{2} & 0 \\ 0 & \frac{\rho_{26}^{A_1 A_2 B}}{\sqrt{2}} & \frac{\rho_{26}^{A_1 A_2 B}}{\sqrt{2}} & 0 & 0 & 0 & 0 & \rho_{66}^{A_1 A_2 B} \end{pmatrix}, \quad (17)$$

with the matrix elements given by

$$\begin{aligned}\rho_{11}^{A_1 A_2 B}(\tau) &= \left( \frac{1}{\cosh s} \right)^2 \sum_{n=0k, l=0}^{\infty} \sum^n (\tanh s)^{2n} G_{kl}^{nn}(\theta) \cos^2 \left( \tau \sqrt{n-l} \right) \\ &\quad \left( \frac{n-k-1}{2n-2k-1} + \frac{n-k}{2n-2k-1} \cos(f_{nk}\tau) \right)^2,\end{aligned}\quad (18)$$

$$\begin{aligned}\rho_{22}^{A_1 A_2 B}(\tau) &= \left( \frac{1}{\cosh s} \right)^2 \sum_{n=0k, l=0}^{\infty} \sum^n (\tanh s)^{2n} G_{kl}^{nn}(\theta) \cos^2 \left( \tau \sqrt{n-l} \right) \\ &\quad \frac{(n-k)}{(2n-2k-1)} \sin^2(f_{nk}\tau),\end{aligned}\quad (19)$$

$$\begin{aligned}\rho_{33}^{A_1 A_2 B}(\tau) &= \left( \frac{1}{\cosh s} \right)^2 \sum_{n=0k, l=0}^{\infty} \sum^n (\tanh s)^{2n} G_{kl}^{nn}(\theta) \cos^2 \left( \tau \sqrt{n-l} \right) \\ &\quad \frac{(n-k)(n-k-1)}{(2n-2k-1)^2} (\cos(f_{nk}\tau) - 1)^2,\end{aligned}\quad (20)$$

$$\begin{aligned}\rho_{44}^{A_1 A_2 B}(\tau) &= \left( \frac{1}{\cosh s} \right)^2 \sum_{n=0k, l=0}^{\infty} \sum^n (\tanh s)^{2n} G_{kl}^{nn}(\theta) \sin^2 \left( \tau \sqrt{n-l} \right) \\ &\quad \frac{1}{(2n-2k-1)^2} ((n-k-1) + (n-k) \cos(f_{nk}\tau))^2,\end{aligned}\quad (21)$$

$$\begin{aligned}\rho_{55}^{A_1 A_2 B}(\tau) &= \left( \frac{1}{\cosh s} \right)^2 \sum_{n=0k, l=0}^{\infty} \sum^n (\tanh s)^{2n} G_{kl}^{nn}(\theta) \sin^2 \left( \tau \sqrt{n-l} \right) \\ &\quad \frac{(n-k)}{(2n-2k-1)} \sin^2(f_{nk}\tau),\end{aligned}\quad (22)$$

$$\begin{aligned}\rho_{66}^{A_1 A_2 B}(\tau) &= \left( \frac{1}{\cosh s} \right)^2 \sum_{n=0k, l=0}^{\infty} \sum^n (\tanh s)^{2n} G_{kl}^{nn}(\theta) \sin^2 \left( \tau \sqrt{n-l} \right) \\ &\quad \frac{(n-k)(n-k-1)}{(2n-2k-1)^2} (\cos(f_{nk}\tau) - 1)^2,\end{aligned}\quad (23)$$

$$\begin{aligned}\rho_{51}^{A_1 A_2 B}(\tau) &= \left( \frac{1}{\cosh s} \right)^2 \sum_{n=0k, l=0}^{\infty} \sum^n (\tanh s)^{2n+1} G_{kl}^{nn+1}(\theta) \frac{\sqrt{(n-k+1)}}{\sqrt{(2n-2k+1)}} \\ &\quad \cos \left( \tau \sqrt{n-l} \right) \sin \left( \tau \sqrt{n-l+1} \right) \sin(f_{n+1k}\tau) \\ &\quad \left( \frac{(n-k-1)}{(2n-2k-1)} + \frac{(n-k)}{(2n-2k-1)} \cos(f_{nk}\tau) \right),\end{aligned}\quad (24)$$

$$\begin{aligned}\rho_{62}^{A_1 A_2 B}(\tau) &= \left( \frac{1}{\cosh s} \right)^2 \sum_{n=0k, l=0}^{\infty} \sum^n (\tanh s)^{2n+1} G_{kl}^{nn+1}(\theta) \frac{\sqrt{(n-k-1)}}{\sqrt{(2n-2k-1)}} \\ &\quad \frac{\sqrt{(n-k)(n-k+1)}}{(2n-2k+1)} \cos \left( \tau \sqrt{n-l} \right) \\ &\quad \sin \left( \tau \sqrt{n-l+1} \right) (\sin(f_{nk}\tau)) (\cos(f_{n+1k}\tau) - 1),\end{aligned}\quad (25)$$

and  $f_{nk} = \sqrt{2(2n-2k-1)}$ .

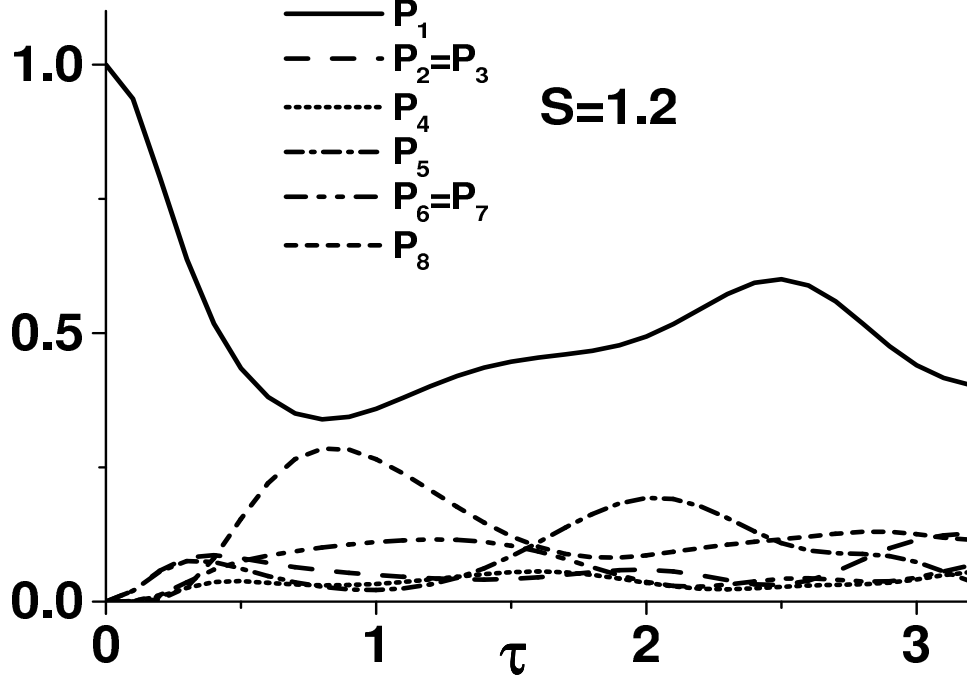


FIG. 2: The probabilities, versus  $\tau (= g\eta t)$  for  $s = 1.2$ .

## V. PROBABILITY OF GENERATING THREE QUBIT ENTANGLED STATES

We have calculated, numerically, the matrix elements of state operator  $\hat{\rho}^{A_1 A_2 B}(\tau)$  using a simple fortran program. The sum  $\sum_{n=0}^{\infty}$  in Eq. (4) is approximated by  $\sum_{n=0}^{n_{\max}}$ , where the value of  $n_{\max}$  is determined by the choice of parameter  $s$ . By analyzing the variation of  $\rho_{ii}^{A_1 A_2 B}(\tau)$  ( $i = 1 - 8$ ) with parameter  $s$  we have chosen  $s = 1.2$  as the preferred value of squeeze parameter and  $n_{\max} = 80$ . A direct observation of  $-10$  dB squeezing of quantum noise of light, reported recently by H. Vahlbruch et al. [14], makes the selected  $s$  value a reasonable choice.

Figure (2) displays the probabilities  $P_i$  ( $i = 1 - 8$ ) of finding the three atoms in the local basis states  $|i_1 i_2\rangle_{c_1} |i_3\rangle_{c_2}$  ( $i = i_1 + 2i_2 + 4i_3 + 1$ ), for  $s = 1.2$  and  $\theta = \pi$ . We notice that the probability of finding the atoms in their ground states drops to 0.34 at  $\tau = 0.8$ , which is already less than  $\sum_{i=2-8} P_i$  at  $\tau \sim 0.4$ . For  $s = 1.2$ , the probability of initial atomic state interacting with the field is reasonably large, while the mean photon number is not too large. In case the mean photon number is large, the cavity decay must be taken into account.

The operator  $\hat{\rho}^{A_1 A_2 B}(\tau)$  of Eq. (17), can be rewritten in diagonal form as

$$\hat{\rho}^{A_1 A_2 B}(\tau) = \sum_{i=1}^6 p_i |\Phi_i\rangle \langle \Phi_i|, \quad (26)$$

where  $p_i$ , and  $|\Phi_i\rangle$  are the eigenvalues and eigenvectors of  $\hat{\rho}^{A_1 A_2 B}(\tau)$  given by

$$\begin{aligned} |\Phi_1\rangle &= a_{000} |00\rangle_{c_1} |0\rangle_{c_2} + a_{101} \left( \frac{|10\rangle_{c_1} + |01\rangle_{c_1}}{\sqrt{2}} \right) |1\rangle_{c_2}, \\ p_1 &= \frac{(\rho_{11}^A + \rho_{55}^A)}{2} - \frac{1}{2} \sqrt{(\rho_{11}^A - \rho_{55}^A)^2 + 4(\rho_{15}^A)^2}, \end{aligned} \quad (27)$$

$$\begin{aligned} |\Phi_2\rangle &= b_{000} |00\rangle_{c_1} |0\rangle_{c_2} + b_{101} \left( \frac{|10\rangle_{c_1} + |01\rangle_{c_1}}{\sqrt{2}} \right) |1\rangle_{c_2}, \\ p_2 &= \frac{(\rho_{11}^A + \rho_{55}^A)}{2} + \frac{1}{2} \sqrt{(\rho_{11}^A - \rho_{55}^A)^2 + 4(\rho_{15}^A)^2}, \end{aligned} \quad (28)$$

$$\begin{aligned}
|\Phi_3\rangle &= c_{100} \left( \frac{|10\rangle_{c_1} + |01\rangle_{c_1}}{\sqrt{2}} \right) |0\rangle_{c_2} + c_{111} |11\rangle_{c_1} |1\rangle_{c_2}, \\
p_3 &= \frac{\rho_{22}^A + \rho_{66}^A}{2} + \frac{1}{2} \sqrt{(\rho_{22}^A - \rho_{66}^A)^2 + 4\rho_{26}^2},
\end{aligned} \tag{29}$$

$$\begin{aligned}
|\Phi_4\rangle &= d_{100} \left( \frac{|10\rangle_{c_1} + |01\rangle_{c_1}}{\sqrt{2}} \right) |0\rangle_{c_2} + d_{111} |11\rangle_{c_1} |1\rangle_{c_2}, \\
p_4 &= \frac{\rho_{22}^A + \rho_{66}^A}{2} - \frac{1}{2} \sqrt{(\rho_{22}^A - \rho_{66}^A)^2 + 4\rho_{26}^2},
\end{aligned} \tag{30}$$

$$\begin{aligned}
|\Phi_5\rangle &= |11\rangle_{c_1} |0\rangle_{c_2}, & p_5 &= \rho_{33}^A, \\
|\Phi_6\rangle &= |00\rangle_{c_1} |1\rangle_{c_2}, & p_6 &= \rho_{44}^A.
\end{aligned} \tag{31}$$

The coefficients  $a_{000}$ ,  $a_{101}$ ,  $b_{000}$ ,  $b_{101}$ ,  $c_{100}$ ,  $c_{111}$ , and  $d_{100}$ ,  $d_{111}$  are known in terms of the matrix elements of  $\hat{\rho}^{A_1 A_2 B}(\tau)$ . The probabilities  $p_i$  determine the entanglement distribution amongst the three qubits in the system. The states  $|\Phi_i\rangle$ ,  $i = 1$  to 4 are W-like states. The state  $\hat{\rho}^{A_1 A_2 B}(\tau)$  is a mixture of W-like states. For example, the fidelity of projecting out the state

$$|W_1\rangle = \frac{1}{\sqrt{3}} (|00\rangle_{c_1} |0\rangle_{c_2} + |10\rangle_{c_1} |1\rangle_{c_2} + |01\rangle_{c_1} |1\rangle_{c_2}), \tag{32}$$

is given by

$$F(W_1) = \text{Tr} (\hat{\rho}^{A_1 A_2 B}(\tau) |W_1\rangle \langle W_1|). \tag{33}$$

Alice can perform a controlled not gate on qubits  $A_1$  and  $A_2$  transforming the state into a mixture of GHZ like states.

It is easily verified that the probability of projecting out the qubits  $A_1 A_2$  in state  $\left( \frac{|10\rangle_{c_1} + |01\rangle_{c_1}}{\sqrt{2}} \right)$  from the reduced state  $\hat{\rho}^{A_1 A_2}(\tau) = \text{Tr}_B (\hat{\rho}^{A_1 A_2 B}(\tau))$  is  $(\rho_{11}^A + \rho_{55}^A)$ .

## VI. THREE QUBIT MIXED STATE ENTANGLEMENT DYNAMICS

The global partial transpose [7] of  $\hat{\rho}^{A_1 A_2 B}(\tau)$  with respect to sub-systems,  $A_1$ ,  $A_2$ , or  $B$  is obtained by using the relations

$$\begin{aligned}
\langle i_1 i_2 i_3 | \rho_G^{T_{A_1}}(\tau) | j_1 j_2 j_3 \rangle &= \langle j_1 i_2 i_3 | \hat{\rho}^{A_1 A_2 B}(\tau) | i_1 j_2 j_3 \rangle, \\
\langle i_1 i_2 i_3 | \rho_G^{T_{A_2}}(\tau) | j_1 j_2 j_3 \rangle &= \langle i_1 j_2 i_3 | \hat{\rho}^{A_1 A_2 B}(\tau) | j_1 i_2 j_3 \rangle, \\
\langle i_1 i_2 i_3 | \rho_G^{T_B}(\tau) | j_1 j_2 j_3 \rangle &= \langle i_1 i_2 j_3 | \hat{\rho}^{A_1 A_2 B}(\tau) | j_1 j_2 i_3 \rangle.
\end{aligned} \tag{34}$$

The  $K$ -way partial transpose [12] ( $K = 2, 3$ ) of tripartite state  $\hat{\rho}^{A_1 A_2 B}$  with respect to qubits  $A_1$ ,  $A_2$ , or  $B$  is constructed by applying the following constraints:

$$\begin{aligned}
\langle i_1 i_2 i_3 | \hat{\rho}_K^{T_{A_1}} | j_1 j_2 j_3 \rangle &= \langle j_1 i_2 i_3 | \hat{\rho}^{A_1 A_2 B} | i_1 j_2 j_3 \rangle, & \text{if } \sum_{m=1}^3 (1 - \delta_{i_m, j_m}) &= K, \\
\langle i_1 i_2 i_3 | \hat{\rho}_K^{T_{A_1}} | j_1 j_2 j_3 \rangle &= \langle i_1 i_2 i_3 | \hat{\rho}^{A_1 A_2 B} | j_1 j_2 j_3 \rangle & \text{if } \sum_{m=1}^3 (1 - \delta_{i_m, j_m}) &\neq K,
\end{aligned} \tag{35}$$

$$\begin{aligned}
\langle i_1 i_2 i_3 | \hat{\rho}_K^{T_{A_2}} | j_1 j_2 j_3 \rangle &= \langle i_1 j_2 i_3 | \hat{\rho}^{A_1 A_2 B} | j_1 i_2 j_3 \rangle, & \text{if } \sum_{m=1}^3 (1 - \delta_{i_m, j_m}) &= K, \\
\langle i_1 i_2 i_3 | \hat{\rho}_K^{T_{A_2}} | j_1 j_2 j_3 \rangle &= \langle i_1 i_2 i_3 | \hat{\rho}^{A_1 A_2 B} | j_1 j_2 j_3 \rangle & \text{if } \sum_{m=1}^3 (1 - \delta_{i_m, j_m}) &\neq K,
\end{aligned} \tag{36}$$

and

$$\begin{aligned} \langle i_1 i_2 i_3 | \hat{\rho}_K^{T_B} | j_1 j_2 j_3 \rangle &= \langle i_1 i_2 j_3 | \hat{\rho}^{A_1 A_2 B} | j_1 j_2 i_3 \rangle, \quad \text{if} \quad \sum_{m=1}^3 (1 - \delta_{i_m, j_m}) = K, \\ \langle i_1 i_2 i_3 | \hat{\rho}_K^{T_B} | j_1 j_2 j_3 \rangle &= \langle i_1 i_2 i_3 | \hat{\rho}^{A_1 A_2 B} | j_1 j_2 j_3 \rangle \quad \text{if} \quad \sum_{m=1}^3 (1 - \delta_{i_m, j_m}) \neq K, \end{aligned} \quad (37)$$

where  $\delta_{i_m, j_m} = 1$  for  $i_m = j_m$ , and  $\delta_{i_m, j_m} = 0$  for  $i_m \neq j_m$ .

The negative part of  $\hat{\rho}_G^{T_p}$  ( $p = A_1$  or  $A_2$  or  $B$ ) contains information about the amount and nature of entanglement present in the tripartite system. By using a  $K$ -way partial transpose one can extract the quantity of  $K$ -way coherence (that is the quantum correlations present in a  $K$ -qubit GHZ like state) in the three qubit state. The contribution of a  $K$ -way partial transpose to the global negativity is calculated by noting that the global partial transpose with respect to qubit  $p$ , may be written as

$$\hat{\rho}_G^{T_p} = \hat{\rho}_3^{T_p} + \hat{\rho}_2^{T_p} - \hat{\rho}. \quad (38)$$

Using  $\text{Tr}(\hat{\rho}_G^{T_p}) = 1$ , the negativity of  $\hat{\rho}_G^{T_p}$  is given by

$$N_G^p = -2 \sum_i \langle \Psi_i^{G-} | \hat{\rho}_G^{T_p} | \Psi_i^{G-} \rangle = -2 \sum_i \lambda_i^{G-}, \quad (39)$$

where  $\lambda_i^{G-}$  and  $|\Psi_i^{G-}\rangle$  are, respectively, the  $i^{\text{th}}$  negative eigenvalue and eigenvector of  $\hat{\rho}_G^{T_p}$ . Substituting Eq. (38) in Eq. (39), we get

$$-2 \sum_i \lambda_i^{G-} = -2 \sum_{K=2}^N \sum_i \langle \Psi_i^{G-} | \hat{\rho}_K^{T_p} | \Psi_i^{G-} \rangle + 2 \sum_i \langle \Psi_i^{G-} | \hat{\rho} | \Psi_i^{G-} \rangle. \quad (40)$$

Defining partial  $K$ -way negativity  $E_K^p$  ( $K = 2$  to  $N$ ) as

$$E_K^p = -2 \sum_i \langle \Psi_i^{G-} | \hat{\rho}_K^{T_p} | \Psi_i^{G-} \rangle, \quad \text{while} \quad E_0^p = -2 \sum_i \langle \Psi_i^{G-} | \hat{\rho} | \Psi_i^{G-} \rangle, \quad (41)$$

we may split the global negativity for qubit  $p$  as

$$N_G^p = E_3^p + E_2^p - E_0^p. \quad (42)$$

The necessary condition for an  $N$ -partite pure state not to have  $N$ -partite entanglement is that the global negativity is zero for at least one of the subsystems that is  $N_G^p = 0$ , where  $p$  is one part in a bipartite split of the composite system.

The global negativity of partial transpose of  $\hat{\rho}^{A_1 A_2 B}(\tau)$  with respect to  $B$  is found to be

$$N_G^B = -2 (\lambda_1^- + \lambda_2^-), \quad (43)$$

where

$$\lambda_1^- = \left( \frac{\rho_{33}^{B A_1 A_2} + \rho_{55}^{B A_1 A_2}}{2} \right) - \frac{1}{2} \sqrt{\left( \rho_{33}^{B A_1 A_2} - \rho_{55}^{B A_1 A_2} \right)^2 + 4 \left( \rho_{26}^{B A_1 A_2} \right)^2},$$

if

$$\left( \rho_{33}^{B A_1 A_2} + \rho_{55}^{B A_1 A_2} \right) < \sqrt{\left( \rho_{33}^{B A_1 A_2} - \rho_{55}^{B A_1 A_2} \right)^2 + 4 \left( \rho_{26}^{B A_1 A_2} \right)^2}, \quad (44)$$

and zero otherwise. Similarly

$$\lambda_2^- = \left( \frac{\rho_{22}^{B A_1 A_2} + \rho_{44}^{B A_1 A_2}}{2} \right) - \frac{1}{2} \sqrt{\left( \rho_{22}^{B A_1 A_2} - \rho_{44}^{B A_1 A_2} \right)^2 + 4 \left( \rho_{15}^{B A_1 A_2} \right)^2},$$



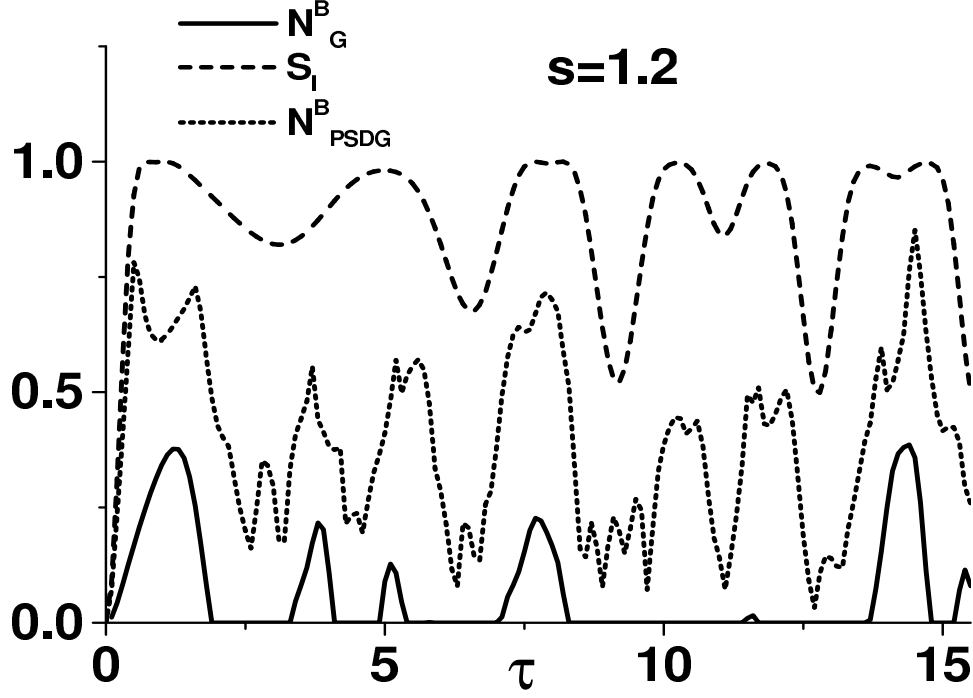


FIG. 3: The global negativity  $N_G^B$ , linear entropy  $S_l$ , and negativity  $N_{PSDG}^B$ , versus  $\tau (= g\eta t)$  for  $s = 1.2$ .

if

$$\left(\rho_{22}^{BA_1A_2} + \rho_{44}^{BA_1A_2}\right) < \sqrt{\left(\rho_{22}^{BA_1A_2} - \rho_{44}^{BA_1A_2}\right)^2 + 4\left(\rho_{15}^{BA_1A_2}\right)^2}, \quad (45)$$

and  $\lambda_2^- = 0$ , if

$$\left(\rho_{22}^{BA_1A_2} + \rho_{44}^{BA_1A_2}\right) \geq \sqrt{\left(\rho_{22}^{BA_1A_2} - \rho_{44}^{BA_1A_2}\right)^2 + 4\left(\rho_{15}^{BA_1A_2}\right)^2}. \quad (46)$$

Furthermore,  $E_3^B = 0$ , implying that the composite system state does not have three qubit GHZ state like coherences.

The reduced state operator for qubit  $B$  is given by

$$\hat{\rho}^B(\tau) = \text{Tr}_{A_1A_2} \left( \hat{\rho}^{A_1A_2B}(\tau) \right). \quad (47)$$

The purity of a state in Hilbert space of dimension  $d$  is measured by the linear entropy defined as

$$S_l(\rho) = \frac{d}{d-1} \left( 1 - \text{tr}(\rho^2) \right). \quad (48)$$

The linear entropy is zero for a pure state and one for a maximally mixed state. Calculated  $S_l^B(\hat{\rho}^B(\tau))$  and  $N_G^B$  are shown as a function of parameter  $\tau (=gt)$  in Figure (3) for  $s = 1.2$  and  $\theta = \pi$ . Whereas mixedness of the state  $\hat{\rho}^B(\tau)$  depends on the entanglement of qubit  $B$  with atomic qubits and cavity field, the negativity of  $\hat{\rho}_G^{TB}(\tau)$  is a measure of entanglement of qubit  $B$  with qubit pair  $A_1A_2$ . We notice that for certain ranges of parameter  $\tau$ ,  $\rho_G^{TB}$  is found to be positive while  $S_l^B$  value indicates that the state is entangled. Can we conclude that for  $N_G^B = 0$ , the qubit  $B$  is entangled only to the cavity field? Since the global negativity is known to detect only free entanglement of mixed states, the answer is no. To detect the bound entanglement, we use the pure state decomposition (PSD) of  $\hat{\rho}^{A_1A_2B}(\tau)$  given by Eq. (26), and calculate

$$N_{PSDG}^p(\hat{\rho}) = \sum_i p_i N_G^p(\hat{\rho}_i), \quad (49)$$

where  $p$  stands for one of the three qubits and subscript  $PSDG$  refers to global negativity calculated for a pure state decomposition. In ref. [8] the convex roof extension of global negativity ( $CRE$ ) has been shown to be en

entanglement monotone capable of detecting bound entanglement. The convex roof extension of negativity for a mixed state is defined as

$$N_{CRE}^p(\hat{\rho}) = \min \left( \sum_i p_i N_G^p(\hat{\rho}_i) \right), \quad (50)$$

where  $\hat{\rho}_i = |\Psi_i(\tau)\rangle \langle \Psi_i(\tau)|$ ,  $|\Psi_i\rangle$  being the eigenvector of  $\hat{\rho}$  corresponding to eigenvalue  $p_i$ . The negativity  $N_{PSDG}^p(\hat{\rho})$ , satisfying the relation

$$N_{PSDG}^p(\hat{\rho}) \geq N_{CRE}^p(\hat{\rho}), \quad (51)$$

is not an entanglement monotone, but serves to detect bound entanglement. Numerically calculated  $N_{PSDG}^B$  for the state  $\hat{\rho}^{A_1 A_2 B}(\tau)$  is also plotted (dotted line) in Figure (3). A comparison with  $N_G^B$  plot shows that the regions in which  $N_G^B = 0$ ,  $N_{PSDG}^B$  is found to be nonzero, implying that it is possible to project out entangled states from  $\hat{\rho}^{A_1 A_2 B}(\tau)$  for any value of  $\tau > 0$ . For  $N_G^B = 0$ , the state of qubit  $B$  is not a separable state in the three qubit mixed state.

For the state of Eq. (26),  $N_3^p(\hat{\rho}_i) = 0$  for all the pure states  $\hat{\rho}_i$  appearing in the pure state decomposition. This is consistent with the earlier observation that the state does not have genuine tripartite entanglement. Genuine tripartite entanglement refers to tripartite entanglement due to correlations similar to those present in a three qubit GHZ-like state. Loss of a single qubit destroys this type of entanglement completely, leaving no residual entanglement. Only two qubit coherences are present in the state  $\hat{\rho}^{A_1 A_2 B}(\tau)$  as evidenced by  $N_G^p(\hat{\rho}_i) = E_2^p(\hat{\rho}_i)$  for all  $\hat{\rho}_i$ . As such we may rewrite the Eq. (49) as

$$N_{PSDG}^p(\hat{\rho}) = \sum_i p_i E_2^p(\hat{\rho}_i). \quad (52)$$

Two qubit coherence can generate not only pairwise entanglement but also W-like tripartite entanglement. It is common practice to trace out subsystem  $A_2$  to obtain the entanglement of the pair  $A_1 B$ . One can, however, obtain a measure of 2-way coherences involving a given pair of subsystems ( $A_1 B$  or  $A_2 B$  or  $A_1 A_2$ ) from 2-way partial transpose constructed from the three qubit state operator  $\hat{\rho}$  by restricting the transposed matrix elements to those for which the state of the third qubit does not change [12]. For example,  $\hat{\rho}_2^{T_{A_1 - A_1 B}}$  is obtained from  $\hat{\rho}$  by applying the condition

$$\begin{aligned} \langle i_1 i_2 i_3 | \hat{\rho}_2^{T_{A_1 - A_1 B}} | j_1 j_2 j_3 \rangle &= \langle j_1 i_2 i_3 | \hat{\rho} | i_1 i_2 j_3 \rangle; \quad \text{if} \quad \sum_{m=1}^3 (1 - \delta_{i_m, j_m}) = 2, \\ \langle i_1 i_2 i_3 | \hat{\rho}_2^{T_{A_1 - A_1 B}} | j_1 j_2 j_3 \rangle &= \langle i_1 i_2 i_3 | \hat{\rho} | j_1 j_2 j_3 \rangle; \quad \text{for all other matrix elements.} \end{aligned} \quad (53)$$

We also define the partial transpose  $\hat{\rho}_2^{T_{A_1 - A_1 A_2}}$ , involving only the pair of subsystems  $A_1 A_2$ , as

$$\langle i_1 i_2 i_3 | \hat{\rho}_2^{T_{A_1 - A_1 A_2}} | j_1 j_2 i_3 \rangle = \langle j_1 i_2 i_3 | \hat{\rho} | i_1 j_2 i_3 \rangle, \quad \text{if} \quad \sum_{m=1}^3 (1 - \delta_{i_m, j_m}) = 2, \quad (54)$$

and for all the matrix elements with  $i_3 \neq j_3$

$$\langle i_1 i_2 i_3 | \hat{\rho}_2^{T_{A_1 - A_1 A_2}} | j_1 j_2 j_3 \rangle = \langle i_1 i_2 i_3 | \hat{\rho} | j_1 j_2 j_3 \rangle. \quad (55)$$

The partial transpose  $\hat{\rho}_2^{T_{B - B A_2}}$  is defined as

$$\langle i_1 i_2 i_3 | \hat{\rho}_2^{T_{B - B A_2}} | i_1 j_2 j_3 \rangle = \langle i_1 i_2 j_3 | \hat{\rho} | i_1 j_2 i_3 \rangle, \quad \text{if} \quad \sum_{m=1}^3 (1 - \delta_{i_m, j_m}) = 2, \quad (56)$$

and for all the matrix elements with  $i_1 \neq j_1$

$$\langle i_1 i_2 i_3 | \hat{\rho}_2^{T_{B - B A_2}} | j_1 j_2 j_3 \rangle = \langle i_1 i_2 i_3 | \hat{\rho} | j_1 j_2 j_3 \rangle. \quad (57)$$

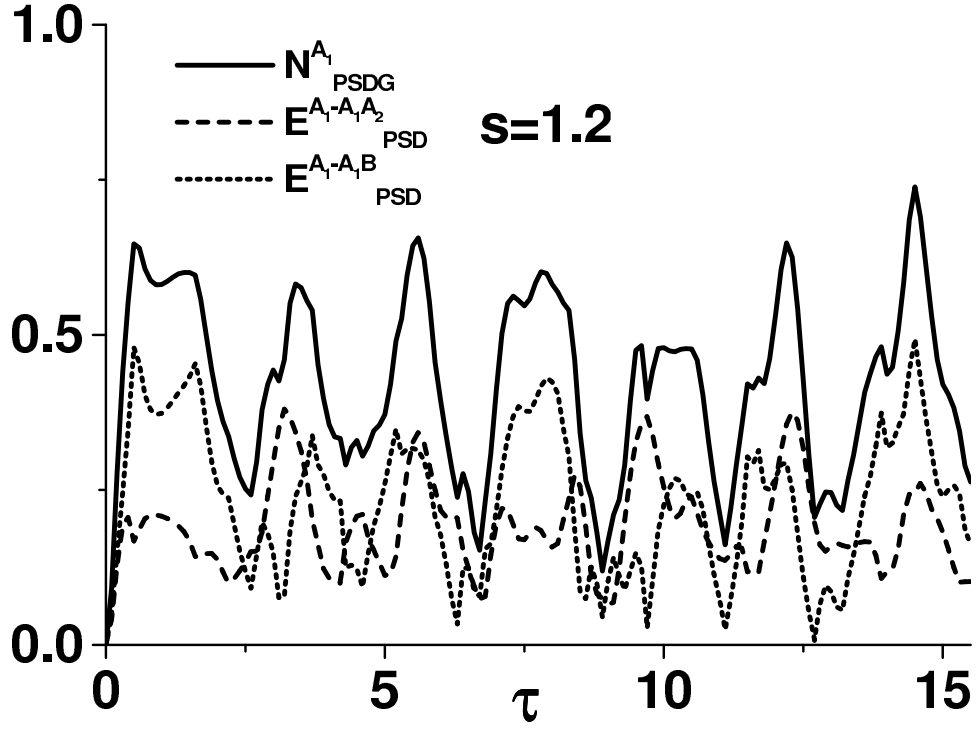


FIG. 4: The negativity  $N_{PSDG}^{A_1}$ ,  $E_{PSD}^{A_1 A_2}$ , and  $E_{PSD}^{A_1 B}$  versus  $\tau (= g\eta t)$  for  $s = 1.2$ .

One can verify that

$$\hat{\rho}_2^{T_B} = \hat{\rho}_2^{T_{B-BA_1}} + \hat{\rho}_2^{T_{B-BA_2}} - \hat{\rho}, \quad (58)$$

and

$$\hat{\rho}_2^{T_{A_1}} = \hat{\rho}_2^{T_{A_1-BA_1}} + \hat{\rho}_2^{T_{A_1-A_1A_2}} - \hat{\rho}. \quad (59)$$

Using the Eqs. (58) and (59) along with the definition of  $E_2^p$  in Eq. (52) we get

$$N_{PSDG}^B(\hat{\rho}) = E_{PSD}^{B-BA_1}(\hat{\rho}) + E_{PSD}^{B-BA_2}(\hat{\rho}), \quad (60)$$

$$N_{PSDG}^{A_1}(\hat{\rho}) = E_{PSD}^{A_1-A_1A_2}(\hat{\rho}) + E_{PSD}^{A_1-A_1B}(\hat{\rho}), \quad (61)$$

where partial negativity  $E_{PSD}^{B-BA_1}(\hat{\rho})$  or  $E_{PSD}^{B-BA_2}(\hat{\rho})$ , associated with pure state decomposition, is defined as

$$E_{PSD}^{B-BA}(\hat{\rho}) = \sum_i p_i \sum_{\mu} \langle \Phi_{\mu,i}^- | (\hat{\rho}_i)_{2}^{T_{B-BA}} | \Phi_{\mu,i}^- \rangle, \quad (62)$$

and  $\Phi_{\mu,i}^-$  is the  $\mu^{th}$  eigenvector corresponding to negative eigenvalue  $\lambda_{\mu,i}$  of the partial transpose  $(\hat{\rho}_i)_2^{T_B}$ . The partial negativities  $E_{PSD}^{A_1-A_1A_2}(\hat{\rho})$  and  $E_{PSD}^{A_1-A_1B}(\hat{\rho})$  are given by

$$E_{PSD}^{A_1-A_1A_2}(\hat{\rho}) = \sum_i p_i \sum_{\mu} \langle \chi_{\mu,i}^- | (\hat{\rho}_i)_2^{T_{A_1-A_1A_2}} | \chi_{\mu,i}^- \rangle,$$

$$E_{PSD}^{A_1-A_1B}(\hat{\rho}) = \sum_i p_i \sum_{\mu} \langle \chi_{\mu,i}^- | (\hat{\rho}_i)_2^{T_{A_1-A_1B}} | \chi_{\mu,i}^- \rangle,$$

where  $\chi_{\mu,i}^-$  is the  $\mu^{th}$  eigenvector corresponding to negative eigenvalue  $\beta_{\mu,i}$  of the partial transpose  $(\hat{\rho}_i)_2^{T_{A_1}}$ .

The partial  $K$ -way negativities  $E_{PSD}^{B-BA_1}(\hat{\rho})$ ,  $E_{PSD}^{B-BA_2}(\hat{\rho})$  and  $E_2^{A_1-A_1A_2}$  calculated from pure state decomposition of state operator determine the pairwise entanglement of qubit pairs  $BA_1$ ,  $BA_2$ , and  $A_1A_2$ . The entanglement available

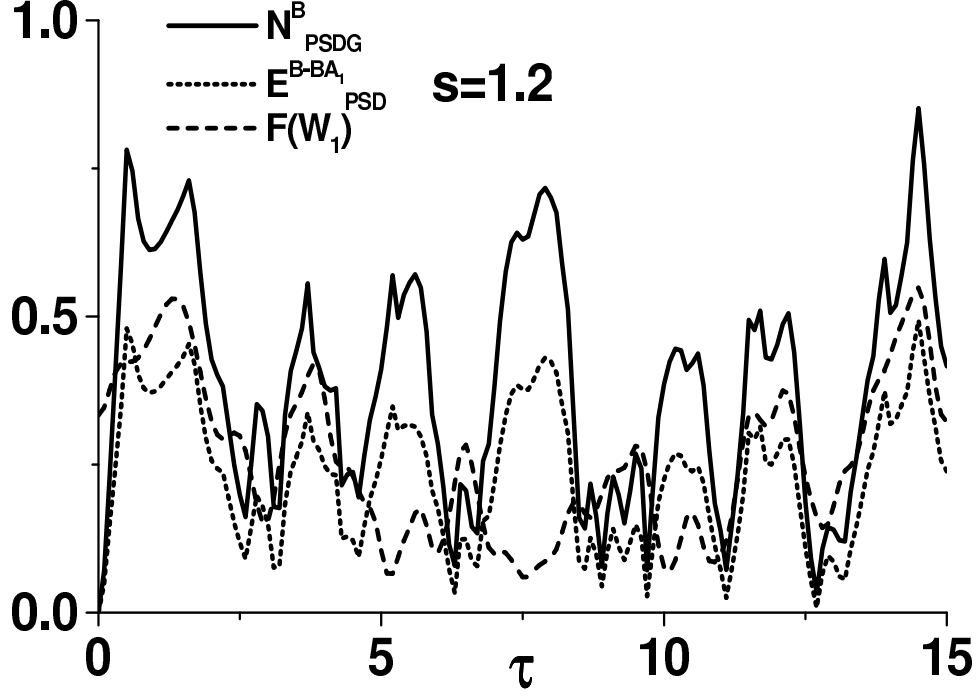


FIG. 5: The negativity  $N_{PSDG}^B$ ,  $E_{PSD}^{BA_1}$  and  $F(W_1)$  versus  $\tau (= g\eta t)$  for  $s = 1.2$ .

to  $A_1$  and  $B$  if they have no knowledge of  $A_2$ , as well as the probabilistic entanglement of subsystem  $BA_1$  after a measurement has been made by  $A_2$ , is determined by  $E_{PSD}^{B-BA_1}(\hat{\rho})$ . The eigenvalues and eigenvectors of  $\hat{\rho}^{A_1A_2B}(\tau)$  are used to calculate, numerically, the partial negativities  $E_{PSD}^{A_1-A_1A_2}(\hat{\rho})$ ,  $E_{PSD}^{A_1-A_1B}(\hat{\rho})$  and  $E_{PSD}^{B-BA_1}(\hat{\rho}) = E_{PSD}^{B-BA_2}(\hat{\rho})$ . Figure (4) displays  $N_{PSDG}^{A_1}(\hat{\rho}) = N_{PSDG}^{A_2}(\hat{\rho})$ ,  $E_{PSD}^{A_1-A_1A_2}(\hat{\rho})$ , and  $E_{PSD}^{A_1-A_1B}(\hat{\rho})$  versus parameter  $\tau$  for  $s = 1.2$  and  $\theta = \pi$ . The negativity  $N_{PSDG}^B(\hat{\rho})$ , and  $E_{PSD}^{B-BA_1}(\hat{\rho}) = E_{PSD}^{B-BA_2}(\hat{\rho})$  are plotted in figure (5) along with the fidelity  $F(W_1)$  defined as in Eq. (33). The maxima in  $F(W_1)$  plot are located in the regions with large  $E_{PSD}^{B-BA_1}(\hat{\rho})$  and finite  $E_{PSD}^{A_1-A_1A_2}(\hat{\rho})$ , consistent with the fact that the reduced two qubit states  $\hat{\rho}^{BA_1}(\tau)$ ,  $\hat{\rho}^{BA_2}(\tau)$ , and  $\hat{\rho}^{A_1A_2}(\tau)$  are entangled states. The partial negativities  $E_{PSD}^{B-BA_1}(\hat{\rho})$ ,  $E_{PSD}^{B-BA_2}(\hat{\rho})$  and  $E_{PSD}^{A_1-A_1A_2}(\hat{\rho})$  detect the pairwise entanglement present in the composite state. Simultaneously nonzero positive valued  $E_{PSD}^{A_1-A_1A_2}$ ,  $E_{PSD}^{A_1-A_1B}$ , and  $E_{PSD}^{B-BA_2}(\hat{\rho})$ , signal the W-like tripartite entanglement of three qubit state  $\hat{\rho}^{A_1A_2B}(\tau)$ .

The calculated negativities  $N_{PSDG}^{A_1}(\hat{\rho})$ ,  $N_{PSDG}^B(\hat{\rho})$ ,  $E_{PSD}^{A_1-A_1B}(\hat{\rho})$  and  $E_{PSD}^{A_1-A_1A_2}(\hat{\rho})$  versus squeeze parameter  $s$  at  $\tau = 14.5$  for  $\theta = \pi$ , are shown in Fig. (6). The decrease in the probability of finding the cavities in vacuum state with increase in the value of  $s$  results in an increase in the probability of finding the system in an entangled state. The peak value of  $N_{PSDG}^B(\hat{\rho})$  occurs for  $s = 0.95$  while  $N_{PSDG}^{A_1}(\hat{\rho})$  is maximized at  $s = 1.04$ . We have selected  $s = 1.2$  as the squeeze parameter for the reason that the probability of finding the qubits  $A_1$ , and  $A_2$  in entangled state is appreciable for this choice.

## VII. CONCLUSIONS

We have studied the entanglement generation through interaction of squeezed light, shared by two remote cavities, with two atoms in cavity  $c_1$  and a single atom in cavity  $c_2$ . No direct interaction amongst the atoms takes place. Analytical expressions for the matrix elements of the three atom mixed state as a function of atom-field coupling strength and interaction time, are obtained. The decrease in the probability of finding the cavities in vacuum state with increase in the value of squeeze parameter  $s$  results in an increase in the probability of finding the system in an entangled state. Numerical calculations to study the dynamics of three atom entanglement show the qubit pair  $A_1A_2$  in Bell-like state entangled to remote qubit  $B$ . From the reduced state operators for three atom system, the linear entropy is calculated and compared with negativities of partially transposed state operators as well as negativities calculated from the pure state decomposition of the state operator. The bound entanglement of the mixed state is not detected by the global negativity, however, the negativity  $N_{PSDG}^B(\hat{\rho})$  calculated from the pure state decomposition of

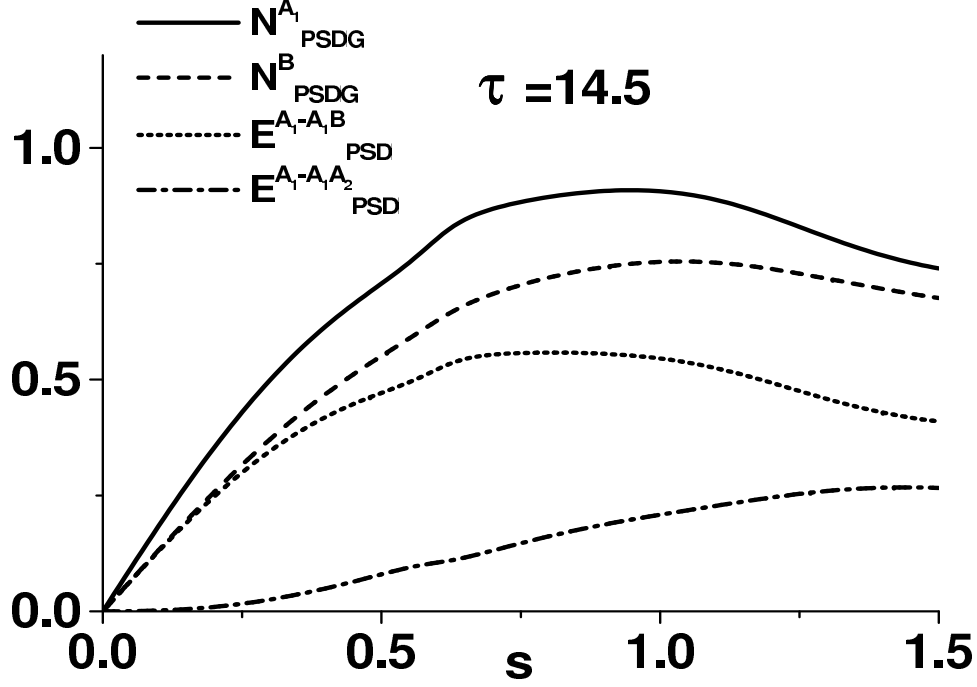


FIG. 6: The pure state decomposition negativities  $N_{PSDG}^{A_1}$ ,  $N_{PSDG}^B$ ,  $E_{PSD}^{A_1A_2}$ , and  $E_{PSD}^{A_1B}$  versus  $s$  for  $\tau = 14.5 (= g\eta t)$

the state operator detects the bound entanglement. The degree of entanglement depends on the squeeze parameter as well as the coefficient of reflection of the beam splitter. For  $\tau = 14.5$ , the pure state decomposition negativity is seen to be maximum for qubit  $A_1$  when  $s = 1.04$ , whereas the peak value of  $N_{PSDG}^B(\hat{\rho})$  occurs for  $s = 0.95$ . A higher value of squeeze parameter,  $s = 1.2$ , has been selected to study the entanglement dynamics of the system since the probability of finding the qubits  $A_1$  and  $A_2$  in entangled state is higher than that for  $s = 1.04$ . The partial negativities calculated by selective partial transposition of the three atom mixed state detect the pairwise entanglement of qubit pairs  $A_1B$ ,  $A_2B$ , and  $A_1A_2$ . The entanglement of three atoms is found to be W-like, no three qubit GHZ like quantum correlations being generated. It is well known, however, that a GHZ state can be distilled from a W-like tripartite state, by stochastic local operations and classical communication. Trapped atoms with long lived metastable states are favored as memory qubits. The advantage of having qubits  $A_1$  and  $A_2$  in Bell-like entangled state is that the entanglement between qubits  $A_1$  and  $B$  can be enhanced through local operations on qubit  $A_2$ . The entangled states, generated in the scheme proposed here, can be used for implementing quantum communication protocols and for quantum computation.

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- [1] A. K. Ekert, Phys. Rev. Lett. 67, 661 (1991).
  - [2] D. Bouwmeester, A. Ekert, and A. Zeilinger, The Physics of Quantum Information (Berlin: Springer) (2000).
  - [3] W. Son, M. S. Kim, J. Lee, and D. Ahn, J. Mod. Opt. 49, 1739 (2002).
  - [4] K. Zyczkowski, P. Horodecki, A. Sanpera, and M. Lewenstein, Phys. Rev. A 58, 883 (1998).
  - [5] A. Peres, Phys. Rev. Lett. 77, 1413(1996).
  - [6] M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Lett. A 223, 1(1996).
  - [7] G. Vidal and R. F. Werner, Phys. Rev. Vol. 65, 032314 (2002).
  - [8] S. Lee, D. P. Chi, S. D. Oh, J. Kim, Phys. Rev. A68 062304 (2003).
  - [9] S. S. Sharma, and N. K. Sharma, arXiv:quant-ph/0608062v2 (unpublished).
  - [10] S. S. Sharma, and N. K. Sharma, Phys. Rev. A 77, 042117 (2008).

- [11] S. S. Sharma, and N. K. Sharma, Phys. Rev. A 76, 012326 (2007).
- [12] S. S. Sharma, and N. K. Sharma, Phys. Rev. A 78, 012113 (2008).
- [13] Tavis and F.W. Cummings, Phys. Rev. 170, 379 (1968).
- [14] H. Vahlbruch, M. Mehmet, S. Chelkowski, B. Hage, A. Franzen, N. Lastzka, S. Goszler, K. Danzmann, and R. Schnabel, PRL 100, 033602 (2008).